

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Mathematics - II

Subject Code: 4SC02MTC1

Branch: B.Sc. (All)

Semester: 2

Date: 31/10/2018

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left[y + 5\left(\frac{dy}{dx}\right)\right]^{\frac{1}{2}}$ is (01)

- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2

b) Solve: $(D^2 - 1)y = 0$ (02)

c) $\int_0^{\pi/2} \cos^4 x \, dx = \underline{\hspace{2cm}}$ (01)

- (a) 0 (b) 1 (c) $\frac{3\pi}{16}$ (d) $\frac{8\pi}{3}$

d) The polar form of $z = 1 - i$ is _____. (01)

- (a) $\sqrt{2}e^{\frac{3\pi}{4}i}$ (b) $\sqrt{2}$ (c) $\sqrt{2}e^{\frac{\pi}{4}i}$ (d) $\sqrt{2}e^{\frac{-\pi}{4}i}$

e) $e^{\pi i} = \underline{\hspace{2cm}}$. (01)

- (a) 0 (b) 1 (c) -1 (d) i

f) Write the equation of Ellipsoid. (01)

g) True/ False: If P.I. = 0 then the general solution of ordinary differential equation is same as the complementary function. (01)

h) Define: Cauchy's sequence. (01)

i) True/ False: Every convergent sequence is bounded. (01)

j) The equation $2(x^2 + y^2 + z^2) - 2xy + 2yz + 2zx = 3a^2$ represents a (01)

- (a) cone (b) sphere (c) right circular cylinder (d) pair of planes

k) Find principal value of $\log(\sqrt{3} - i)$. (02)



1) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = \underline{\hspace{2cm}}$. (01)

(a) 0 (b) 1 (c) $\frac{16}{35}$ (d) $\frac{35\pi}{32}$

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) State and prove De-moivre's theorem (07)

b) Prove that $(1 + \sqrt{3}i)^n + (1 - \sqrt{3}i)^n = 2^{n+1} \cos \frac{n\pi}{3}$. (04)

c) Simplify: $\frac{(\cos 3\theta + i \sin 3\theta)^{-2} (\cos 2\theta - i \sin 2\theta)^3}{(\cos 5\theta - i \sin 5\theta)^3 (\cos 2\theta + i \sin 2\theta)^7}$ (03)

Q-3 Attempt all questions (14)

a) Prove that $\frac{\sin 7\theta}{\sin \theta} = 7 - 56 \sin^2 \theta + 112 \sin^4 \theta - 64 \sin^6 \theta$. (05)

b) Find the roots of the equation $z^4 - 1 = 0$. (05)

c) Solve: $y'' + y' - 2y = e^x + \cos x$ (04)

Q-4 Attempt all questions (14)

a) Using De-moivre's theorem solve $z^7 + z^4 + z^3 + 1 = 0$. (07)

b) Solve: $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 \sin(\log x)$. (07)

Q-5 Attempt all questions (14)

a) Prove that $\cosh^{-1}(z) = \log(z + \sqrt{z^2 - 1})$. (05)

b) Find the principal value of $(-i)^i$. (05)

c) Find the real and imaginary part of $\tanh z$. (03)

Q-6 Attempt all questions (14)

a) Prove that $\int_0^1 \frac{x^6}{(1+x^2)} \, dx$ (05)

b) Evaluate: $\int_0^{\pi} x \cos^6 x \, dx$ (05)

c) Solve: $y'' - 2y' + 4y = e^x \cos x$. (04)

Q-7 Attempt all questions (14)

a) Identify, describe and sketch the surface $16x^2 + 36y^2 + 9z^2 = 144$. (05)

b) Show that the equation $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $(-\frac{7}{6}, \frac{1}{3}, \frac{5}{6})$. (05)



c) Check whether the following sequence convergent or not. (04)

i) $\left\{\frac{1}{n}\right\}$ ii) $\{(-1)^n\}$

Q-8 Attempt all questions (14)

a) Find equation of lines in which the plane $2x + y - z = 0$ cuts the cone (05)

$$4x^2 - y^2 + 3z^2 = 0.$$

b) Find equation of cylinder whose generators are parallel to $\frac{x}{3} = \frac{y}{2} = \frac{z}{1}$ and guiding (05)

$$\text{curve } x^2 + y^2 + z^2 = 9.$$

c) Solve: $(D^7 + 18D^5 + 81D^3)y = 0$ (04)

